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Comparison of Local Geoid Models Using Fast Fourier Transform and Least Squares Collocation in the Remove-Compute-Restore Scheme: A Case Study of Pangkal Pinang

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Abstract

The geoid model is crucial for all nations as the primary height reference. Given that numerous countries have developed their geoid models, local geoid modelling is essential for Indonesia. According to the Indonesian Geospatial Reference System (SRGI 2013), which encompasses the national height reference system, it is indicated that not all areas of Indonesia possess a geoid model with a high degree of accuracy. The geoid modelling in Indonesia has been conducted incrementally. In local geoid modelling utilizing gravimetric techniques, specifically within the Remove Compute Restore (RCR) framework, two approaches for ascertaining residual undulation are the Fast Fourier Transform (FFT) Method and the Least Squares Collocation (LSC) Method. This study examines a comparison of the two methods, focusing on the accuracy of the geoid model and the factors that affect its precision. This study's findings demonstrate that the least squares collocation geoid model is the most precise geoid model in the Pangkal Pinang region and its vicinity, achieving an accuracy of ±0.131 m. The accuracy of the Fast Fourier Transform method is ±0.149 m for the 2D Spherical Multiband FFT approach and ±0.159 m for Haagman's 1D Spherical FFT approach.
Keywords: *pangkal pinang, geoid, remove compute restore, fft, lsc*

Abstrak

Model geoid merupakan hal penting bagi tiap negara sebagai suatu datum tinggi yang paling relevan. Menilik bahwa banyak negara telah memodelkan geoid negaranya, pemodelan geoid lokal menjadi penting bagi negara Indonesia. Berdasarkan Sistem Referensi Geospasial Indonesia (SRGI 2013), yang di dalamnya termasuk sistem referensi tinggi nasional, menyatakan bahwa saat ini belum semua wilayah Indonesia memiliki model geoid yang mempunyai tingkat akurasi yang tinggi. Pemodelan geoid di Indonesia sendiri secara bertahap telah dilakukan. Pada pemodelan geoid lokal secara gravimetrik, khususnya pada skema Remove Compute Restore (RCR), terdapat dua metode dalam penentuan undulasi residual, yaitu Metode *Fast Fourier Transform* (FFT) dan Metode *Least Square Collocation* (LSC). Pada penelitian ini dikaji perbandingan kedua metode ini, khususnya perbandingan mengenai ketelitian model geoid dan faktor yang mempengaruhi ketelitiannya. Hasil dari penelitian ini menunjukkan bahwa model geoid *Least Square Collocation* adalah model geoid yang paling teliti di area Pangkal Pinang dan sekitarnya dengan ketelitian ±0.131 m. Sedangkan ketelitian metode Fast Fourier Transform adalah ±0.149 m untuk pendekatan 2D Spherical Multiband FFT dan ±0.159 m untuk pendekatan Haagman's 1D Spherical FFT.
Kata Kunci: *pangkal pinang, geoid, remove compute restore, fft, lsc*

1. Introduction

Indonesia is the world's largest archipelagic nation, with 17,504 islands. This circumstance renders the unification of the vertical reference system in Indonesia a formidable undertaking. The consolidation of vertical reference systems predicated on mean sea level (MSL) is irrelevant due to the variability of mean sea level throughout different regions of Indonesia [2]. Consequently, the integration of the vertical reference system can be accomplished by developing a local Indonesian geoid model [3]. The Geoid Model is essential for all nations as the primary vertical datum. The geoid model serves as a height datum for accurately estimating the elevation of a point. According to the Indonesian Geospatial Reference System (SRGI 2013), which encompasses the national height reference system, there are presently several geoid models of high precision in Indonesia, such as local geoid modelling in the central part of Java [4] and a high-precision geoid model for Bali [5]. Nonetheless, many places in Indonesia lack a local geoid model.

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Geoid modelling can be conducted geometrically (utilizing GNSS/levelling data) and gravimetrically (employing gravity and GNSS data) [6]. Geoid modeling in Indonesia has been progressively conducted. The authorized agency in Indonesia, the Geospatial Information Agency (BIG), uses the gravimetric approach for local geoid modelling. In local geoid modelling utilizing the gravimetric method, several techniques are recognized, including Remove Compute Restore (RCR) [7] and Least Squares Modification of Stokes (LSMS) [8]. The Remove Compute Restore (RCR) approach is the most commonly employed technique globally, including in Indonesia.

The Remove Compute Restore (RCR) approach is a first-order approximation of the Molodensky theory for estimating the quasi-geoid and integrates Stokes with topographic mass regularization [9]. Remove Compute Restore (RCR) integrates three data elements: long wavelength components derived from global geopotential model data, medium wavelength components sourced from local terrestrial gravity data, and short wavelength components obtained from digital terrain model data. Of the three components, the long wavelength component, or global geopotential model data, gives the greatest value and error. The impact of the other two components is comparatively minimal [10].

The choice of the global geopotential model is a critical aspect of the RCR approach. Based on the research conducted by Pahlevi et al. [11], the EGM2008 model at degree 2190 is the most accurate global geopotential model for the Indonesian region. According to Sabri et al. [12], geoid modelling on the island of Java utilizing the Stokes Integral, or the Remove Compute Restore (RCR) approach, achieves a local geoid model accuracy of ±0.063 m. The Remove Compute Restore (RCR) scheme employs two methods for calculating residual geoid undulation: the Fast Fourier Transform (FFT) method and the Least Squares Collocation (LSC) approach. According to the study conducted by Triarahmadhana et al. [13], the local geoid modelling in Yogyakarta utilizing the FFT approach achieved an accuracy of 0.127 m, surpassing the accuracy target of the BIG Local Geoid Model established at 0.150 m [11]. Both methodologies for ascertaining the residual geoid undulation are critical determinants affecting the precision of the local geoid model. This research examines the accuracy outcomes of both methods, which are essential components in local geoid modelling.

2. Material and Methods

Research Study Area

The study was carried out in the Pangkal Pinang region and its vicinity, defined by coordinates ranging from 2°27' South to 1°51' South and from 105°54' East to 106°18' East, exhibiting moderate topography diversity. The research area comprises three administrative city districts within Bangka Province. To the east, the research area is predominantly delineated by the sea.

Fig. 1: Research study area in Pangkal Pinang

Data

This study utilizes three sorts of data. The EGM2008 degree 2190 data serves as a long-wave component, representing the global geopotential model. EGM2008 offers data up to degree 2190 coefficients. Pahlevi's [11] study asserts that the EGM2008 degree 2190, or maximum degree, is the most effective global geopotential model for use in Indonesia. The secondary dataset utilized is height data, functioning as the short-wave component. Height data is utilized to mitigate the impacts of topography and incorporate topographic compensation in local geoid modeling. The elevation data utilized in this investigation is DEMNAS data sourced from the BIG website. DEMNAS possesses a spatial resolution of 0.6 arcminutes. The third dataset utilized is terrestrial gravity data, which functions as the medium-wave component. The terrestrial gravity data is processed to get free-air gravity anomalies. The free-air gravity anomalies will be utilized as input data for local geoid modeling as a medium-wave component. The terrestrial gravity data was acquired via a survey executed by the Geospatial Information Agency in August 2018 in Pangkal Pinang and its vicinity, encompassing 87 observation locations.

Local Geoid Modeling

Local geoid modeling relies on two fundamental equations: the Bruns Formula and the Stokes Function. The Bruns formula illustrates the correlation between potential anomaly and undulation [14]. Stokes defines the value of the potential anomaly as a function of the gravity anomaly and the spherical distance between a region and a location with a known potential anomaly [15].

This study uses the Remove Compute Restore (RCR) approach for geoid modeling. The removal of compute restore is categorized into three phases: the removal phase, during which the long-wave component (forecasted using a global geopotential model) and the short-wave component (forecasted from a digital terrain model) are extracted from the observed gravity data. During the computation phase, the gravitational anomaly, filtered from both long and short wave components, is converted into a quasi-geoid using Stokes integration. At this juncture, the Fast Fourier Transform (FFT) and Least Squares Collocation (LSC) methodologies are applicable. Subsequent to the compute step, the restore stage entails reintegrating the contributions of the long-wave and short-wave components to the quasi-geoid to construct the geoid. In the RCR approach, the potential anomaly T is segmented into three components, specifically:

$$T = T_{EGM} + T_{RTM} + T_{RES} \quad (1)$$

T_{EGM} represents the input from the global geopotential model (EGM). T_{RTM} represents the impact of the terrain, generally referred to as Residual Terrain Modelling (RTM), while T_{RES} is the residual of the observed gravity field. T is presumed to be a spatial function. One rationale for the diminution of EGM is to depict the gravitational field outside the region encompassed by the data. For terrain reduction, the terrain effect is diminished in relation to the mean surface elevation. The terrain potential is extracted from the observational data by prism integration or rapid Fourier transform.

The prism integration from the RTM implementation possesses an intrinsic issue; specifically, this method permits points above the average surface elevation to remain in the free mass domain, while points below the average surface elevation post-reduction correspond to points within the reference topography mass. The geodesic gravity field modeling method necessitates observations obtained from harmonic functions, including the mass-free environment above the geoid, and applies harmonic corrections to gravity anomaly spots situated below the mean surface elevation [16]. The "restore" geoid signal is computed by prism integration.

During the computation phase, two techniques, FFT and LSC, may be employed. The Fast Fourier Transform (FFT) is a method employed to calculate the Discrete Fourier Transform of a sequence and its inverse. Fourier analysis is commonly employed to convert a signal from its original domain, either time or space, into a frequency domain representation and vice versa. The Fast Fourier Transform pertains to the establishment of the geoid model, frequently employed in terrain rectification (RTM) and the resolution of the Stokes formulation. The assessment of the Stokes formulation can be conducted with FFT, such as with 1-D and 2-D Spheris FFT [17]. The FFT approach significantly enhances the computation of intricate terrain corrections on a broad scale [18]. The FFT algorithm in the assessment of the Stokes formulation is employed to transform residual gravity anomalies into residual geoid undulations. The 2D-FFT Spherical Multiband equation for deriving undulation from the Stokes formulation can be expressed in relation to grid and frequency.

Besides the FFT method, LSC can also be employed in the determination of the quasi-geoid. The fundamental premise of least squares adjustment is to identify a model that minimizes the sum of the squares of the discrepancies of the observed values. A system of linear equations of uniform order for each quantity of observed data is established for the least squares adjustment. Least Squares Collocation (LSC) is

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commonly employed in physical geodesy to address boundary value difficulties in gravimetry. **Figure 2** displays the representation of the local geoid model.

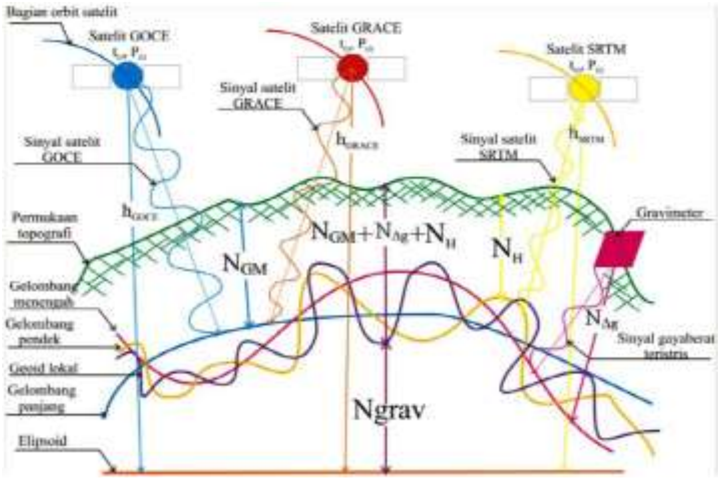

Fig. 2: Illustration of the local geoid model
Source: Triarahmadhana and Heliani (2014)

A number of stages were involved in the execution of this research. The stages are broken down within the flowchart that can be found in **Figure 3**.

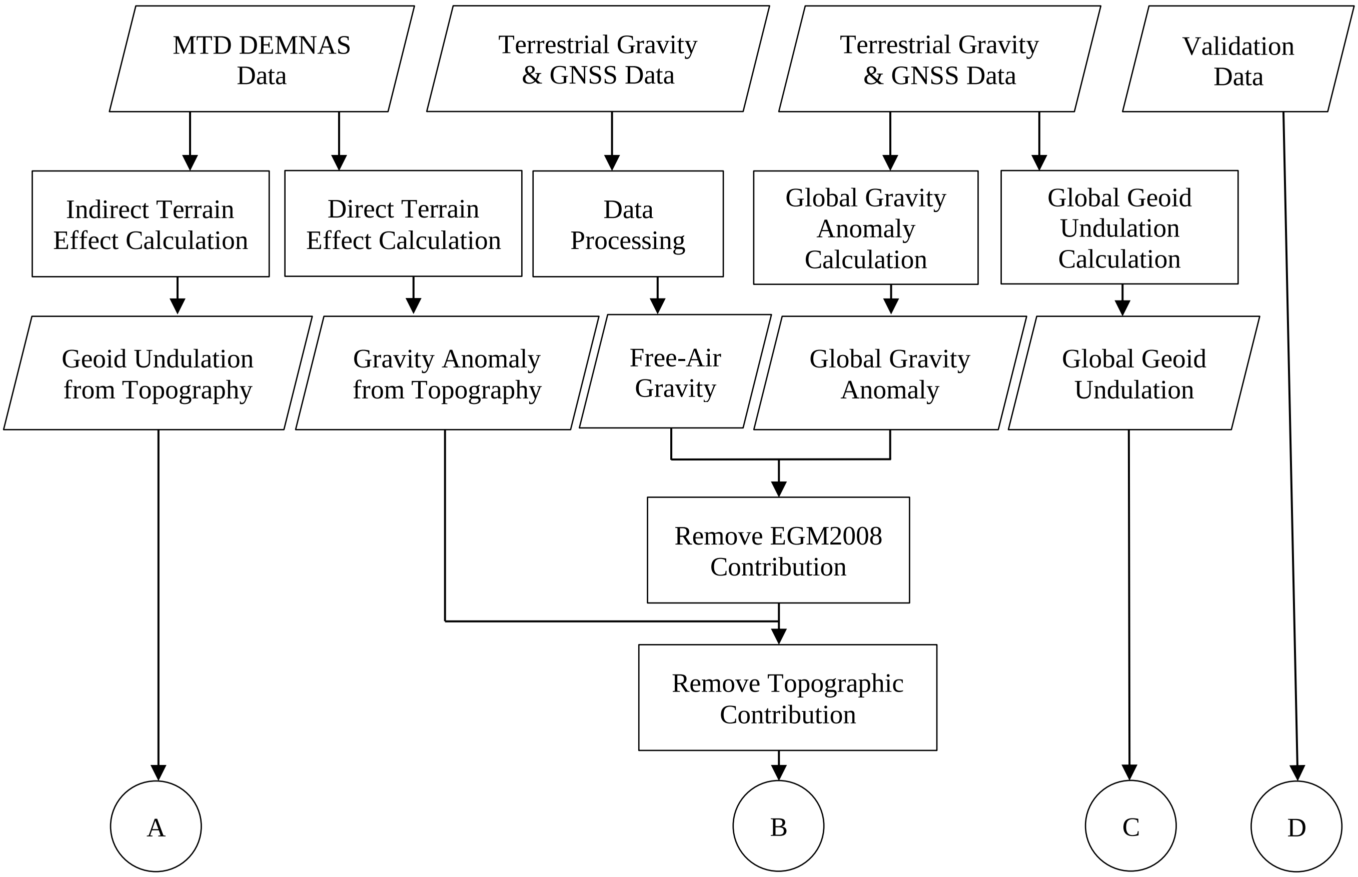

Fig. 3: Diagram of the research process

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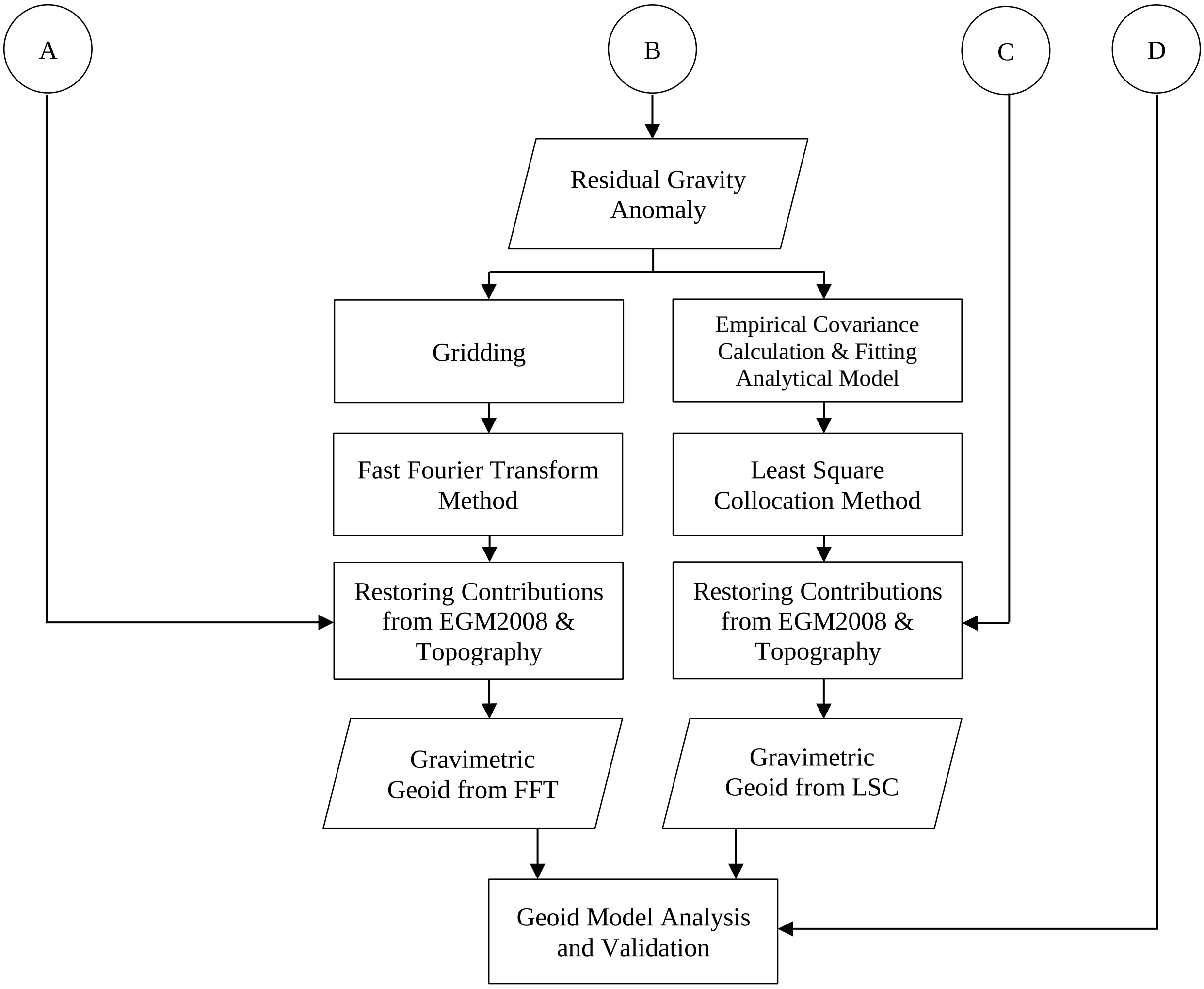

Fig. 3: Diagram of the research process

The phases of research commence with preparation, succeeded by data collection. This data collection encompasses phases of acquiring information from multiple sources. The DEMNAS data was sourced from the DEMNAS BIG website, terrestrial gravity and GNSS data were acquired from BIG, and the EGM2008 data was retrieved from the ICGEM website. Subsequently, the data undergoes processing. The processing of DEMNAS yields direct and indirect effects, while terrestrial gravity data and GNSS generate free-air gravity anomalies; EGM2008 provides global gravity anomalies and global geoid undulations. The removal scheme process is conducted to diminish the free-air gravity anomaly by accounting for the contributions of EGM2008 and topography, yielding a residual anomaly.

The residual anomaly undergoes two distinct phases: gridding, which leads to the FFT stage, and empirical covariance calculation and analytical model fitting, which progresses to the LSC stage. In the FFT process, complete zero padding is implemented to mitigate the effects of periodicity or circular convolution. The calculation of empirical covariance is conducted utilizing a correlation distance of 0.5 arcminutes. The FFT and LSC processes generate residual undulations, which are subsequently rectified by reintroducing the contributions of EGM2008 and topography, culminating in the 1DFFT, 2DFFT, and LSC Geoid models. The three geoid models are validated using two validation points, and the outcomes of this validation will be addressed in the results analysis.

3. Results and Discussion

This study presents gravimetric geoid models derived from the 1DFTT, 2DFFT, and LSC methods, illustrated in Figure 4. The image reveals discrepancies in the geoid model values in the eastern section of the research area. The statistics for the three models are presented in **Table 1**.

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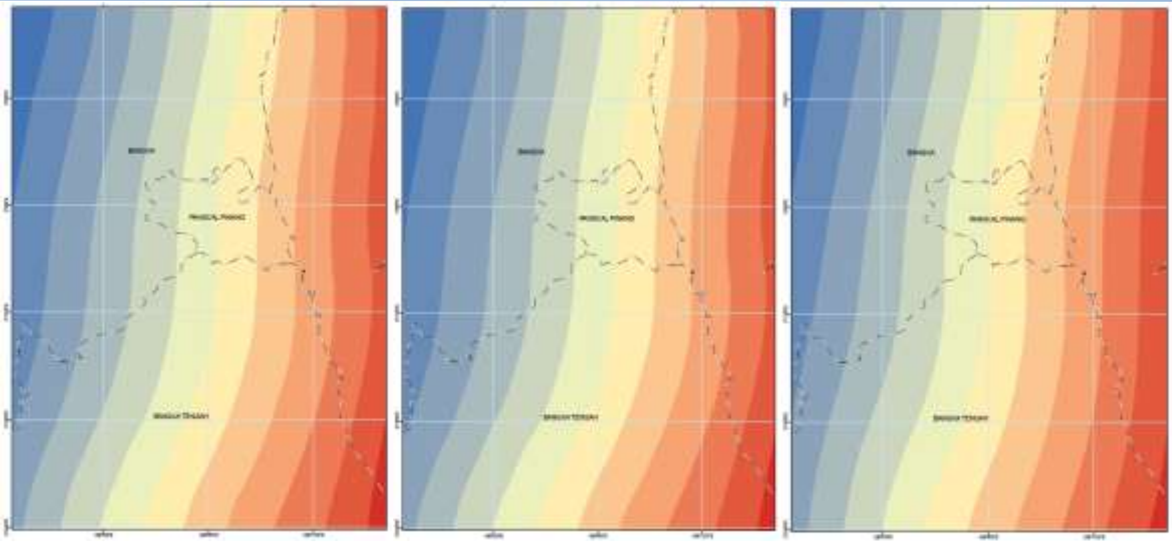

(a) (b) (c)

Fig. 4: Results of the 1DFFT (a), 2DFFT (b), and LSC (c) Geoid Models

The local geoid model generated by the FFT method is contingent upon various factors. The primary influential factors of the FFT method are the gridding effect and the periodicity effect. Gridding to achieve a more accurate local geoid model is accomplished through the use of weighted means. The enhancement in precision with weighted means gridding relative to least square collocation gridding is ±0.01 m. The periodicity effect can be mitigated by implementing 100% zero padding. The implementation of 100% zero padding can enhance the accuracy of the FFT geoid model by ±0.079 m for 2DFFT and ±0.048 m for 1DFFT.

Several factors influence the local geoid model generated by the LSC method. The primary determinant of the LSC method's efficacy is the accessibility of ample data characterized by uniform distribution and minimal variance. The subsequent critical step involves aligning empirical covariance with an analytical model. The establishment of a correlation distance of 0.5 arcminutes in the empirical covariance computation yields an optimal model with a total RMSE of 0.067 $mgal^2$. The optimal fitting model yielded a parameter D (Distance to the Bjerhammer Sphere) of -570.31 m, a long-wave attenuation factor of 0.9062 km, and a variance at the mean elevation of 36.73 $mgal^2$. These three parameters produce a locally accurate geoid model with an uncertainty of approximately ±0.31 m.

Table 1. Statistics for each gravimetric geoid model

	1DFFT (m)	2DFFT (m)	LSC (m)
MAX	19.044	18.954	19.092
MIN	17.488	17.498	17.442
MEAN	18.265	18.262	18.217
STD	0.408	0.408	0.395

The geoid model generated from this research was analyzed comparatively with other models. The comparison of the three models reveals that the 1DFFT Geoid Model and the 2DFFT Geoid Model exhibit minimal variance, with an average discrepancy of 8 mm and a standard deviation of ±4 mm. The disparity between the 1DFFT Geoid Model and the LSC Geoid Model is characterized by an average of 8.2 cm and a standard deviation of ±2.2 cm. The disparity between the 2DFFT Geoid Model and the LSC Geoid Model is presented with an average of 7.9 mm and a standard deviation of ±1.8 cm. **Table 2** presents the comparative statistics of the geoid models. The comparison results were analyzed using a Student's t-test at a 95% confidence level to assess the significance of the differences between the methods. The findings demonstrate that the 1DFFT, 2DFFT, and LSC Geoid Models exhibit negligible differences.

Table 2. Comparative Analysis of Geoid Models

	1DFFT-2DFFT (m)	1DFFT-LSC (m)	2DFFT-LSC (m)
MAX	0.016	0.118	0.118
MIN	0	0.019	0.006
MEAN	0.008	0.048	0.045
STD	0.004	0.021	0.026

The outcomes of the gravimetric geoid model derived from the three methodologies—2DFFT, 1DFFT, and LSC—were subsequently validated at two points in Pangkal Pinang. The outcomes of this validation are presented in Table 3. The findings indicate that the LSC geoid model exhibits the highest gravimetric geoid precision, with a root mean square error (RMSE) of ±0.131 m. The findings align with Łyszkowicz's [19] research, which indicates that the internal accuracy of the LSC method surpasses that of the FFT method. Nevertheless, in regions with restricted data availability and in peripheral areas (outer accuracy), the FFT method will produce superior accuracy. The RMSEs for the 2DFFT and 1DFFT gravimetric geoid models are ±0.149 m and 0.159 m, respectively. Both FFT methods exhibit a discrepancy of ±0.01 m. The enhanced accuracy of the 2DFFT geoid model, with a precision of ± 0.079 m, may be attributed to the zero padding process, which surpasses the accuracy of the 1DFFT geoid model.

Table 3. Validation Results of Geoid Model Accuracy

Point	N (m)	N 2DFFT (m)	N 1DFFT (m)	N LSC (m)
N.1089	18.160	18.339	18.351	18.323
GBU049	18.265	18.378	18.384	18.356
RMSE		0.149	0.159	0.131

4. Conclusion

The geoid model generated from this research exhibits an accuracy of ±0.159 m for the 1DFFT model, ±0.149 m for the 2DFFT model, and ±0.131 m for the LSC model. Consequently, it can be inferred that the internal accuracy of the LSC geoid model in the Pangkal Pinang study area and its vicinity is superior to that of the other models. Nonetheless, there is no substantial difference among the three models following the execution of a t-student test at a 95% confidence level.

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